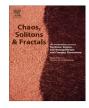
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# Particle swarm optimization for automatic creation of complex graphic characters



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## ABSTRACT

Nature-inspired algorithms are a very promising tool for solving the hardest problems in computer sciences and mathematics. These algorithms are typically inspired by the fascinating behavior at display in biological systems, such as bee swarms or fish schools. So far, these algorithms have been applied in many practical applications. In this paper, we present a simple particle swarm optimization, which allows automatic creation of complex two-dimensional graphic characters. The method involves constructing the base characters, optimizing the modifications of the base characters with the particle swarm optimization algorithm, and finally generating the graphic characters from the solution. We demonstrate the effectiveness of our approach with the creation of simple snowman, but we also outline in detail how more complex characters can be created.

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# 1. Introduction

Computer games are one of the most profitable products of the interactive entertainment industry. Worldwide, the number of active users is staggering, and this largely regardless of age, gender and ethnicity. The positive trend in the number of user since the 80's has been spurred on further with the advent of massive online multi-player games. The demand on the technology to deliver ever more authentic and exhilarating user experience is of course also increasing, and this creates a continuous need for innovation and improvement at many different levels of the industry. In addition to the obvious importance of hardware improvement, the development of new software,

http://dx.doi.org/10.1016/j.chaos.2014.12.019 0960-0779/© 2015 Elsevier Ltd. All rights reserved. new interaction possibilities, as well as a firm positioning in the market all demand expertise and skill. At the juncture of these different aspects of development, theoretical research in computer science, applied mathematics and complex systems also plays a key role. And it is to the latter subject in particular that we aim to contribute with this paper.

In the past decades, the methods and techniques employed first in computer games have been applied with notable success in computational agent-based system research [21]. Many studies have also looked at video games as an art form on its own right [8]. At some universities and research institutes, computer games have become the subject on which one could do a masters or a PhD. Most notably, however, video games have had the most significant impact on the development of personal computers. Due to popular demand, personal computers had to improve steadily in the amount of RAM, processor

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speed, display resolution and graphic card capabilities to keep with the pace of more and more sophisticated games.

Simultaneously with the development of hardware, software methods have also arisen for solving the hardest real-world problems. As many of these problems are NPhard [19], many heuristical algorithms have been proposed for solving such problems approximately. The majority of these heuristics take inspiration from nature. Two branches of algorithms are particularly known for their close ties with nature, namely the evolutionary algorithms (EA) [9] and swarm intelligence algorithms (SI) [4]. The former are inspired by the Darwinian principle of "only the fittest survive" [26], while the later take from social insects, swarms of birds and flocks of fish to execute complex actions based on elementary and indeed very simple interactions among individuals [38]. An intersection of both is the emergence of collective behavior in evolutionary games [28].

In general, it has been established that using advanced nature-inspired algorithms in computer graphics can be very useful. Applications to this domain include the EA in [33,34], where genetic algorithms were introduced to computer graphics. Later on, other evolutionary algorithms were also applied [14,2], while the most recent studies include also genetic algorithms [18] for polynomial Bspline surface reconstruction, particle swarm optimization [16] for the same problem and particle swarm optimization for Bézier surface reconstruction [15]. For example, Gálvez and Iglesias [17] applied the firefly algorithm [10,11] for polynomial Bézier surface parameterization, which is a well known problem in computer graphics. Further to these advances, creative 3D shape modeling has been proposed, where an initial population of 3D models is evolved to produce novel shapes in the next generation [40]. A new emerging field termed search-based procedural content generation is also growing rapidly, the aim of which is to use metaheuristics in automatic generation of content for games [36].

Since many games operate with a large number of different characters, we here address a very basic problem in a new way. In particular, we explore the usefulness of particle swarm optimization (PSO) for the automatic creation of complex graphic characters. We show how a modified particle swarm optimization algorithm can be used to create a gallery of 2D characters with different colors and shapes using a predefined set of parts. For the approach to work, the problem is defined as constraint satisfaction problem (CSP). Each candidate solution that satisfies all constraints is placed in a gallery from which the best characters can be selected by the game designer. As an example, we show the generation of 2D snowman, but we also outline the approach for the creation of 3D and more complex characters.

The remainder of this paper is structured as follows. Section 2 reviews the basics of the constraint satisfaction problem. In Section 3, we present particle swarm optimization algorithm for the automatic creation of graphic characters. The experiments and results are presented in Section 4. We conclude with a summary and directions for future development, as well as with a broader discussion about the relevance of nature-inspired algorithms for the emergence of intelligent design and cognition [29].

# 2. Constraint satisfaction problems

The majority of intractable (NP-hard) real-world problems are constrained [19]. That means, not all of candidate solutions obtained in EA and SI are valid after performing various variation operators. Usually, these operators act blindly according to constraints, i.e., these do not provide valid solutions. The constraint satisfaction problems (CSP) consist of three components  $\langle \mathbf{x}, \mathbf{D}, \mathbf{C} \rangle$  [30], where.

- **x** is a vector of variables  $\mathbf{x} = (x_1, \dots, x_n)$ , where *n* denotes a dimension of the problem,
- **D** is a set of domains **D** = {*D*<sub>1</sub>,...,*D*<sub>n</sub>} for each variable of **x**,
- **C** is a set of constraints **C** = {*C*<sub>1</sub>,...,*C*<sub>m</sub>} that specify allowable combinations of values.

Each domain is determined by its lower and upper bounds  $D_i \in [lb_i, ub_i]$ . Each constraint  $C_i$  consists of a pair  $\langle scope, relation \rangle$ , where *scope* determines variables that participate in the constraint and *relation* defines an relation that needs to be valid in order to satisfy the so named feasibility condition  $\phi$ . The feasibility condition for each candidate solution is composed of constraints and can be expressed as conjunction of constraints  $\phi(\mathbf{x}) = \wedge_{i=1...m}C_i(\mathbf{x})$ . In fact, the feasibility condition is true if and only if all constraints are satisfied.

There are two main types of constraints handling in EA and SI [9]:

- Indirect, where constraints are transformed into optimization objectives,
- Direct, where the population based algorithms keep alone account about constraints during the run.

In this paper, indirect constraint handling was considered, where the proper solution is found, when the feasibility condition is satisfied. In this case, the value of fitness function is zero, i.e.,  $\phi(\mathbf{x}) = true$  if and only if  $f(\mathbf{x}) = 0$ .

### 3. Automatic creation of graphic characters

Particle swarm optimization (PSO) was one of the oldest swarm intelligence algorithms that was introduced at an International conference on Neural Networks by Kennedy and Eberhart in 1995 [23]. PSO is inspired by the social foraging behavior of some animals such as flocking behavior of birds and schooling behavior of fish. There are some individuals with better developed instinct for finding food, in both animal species. According these individuals, the whole swarm is directed into more promising regions in the fitness landscape.

The PSO is a population-based algorithm that consists of particles  $\mathbf{x}_i^{(t)} = (x_{i,1}, \dots, x_{i,n})^T$  for  $i = 1 \dots Np$  representing their position in a *n*-dimensional search space. Thus, the variable Np limits the number of particles in the population. The particles move across the search space with velocity  $\mathbf{v}_i^{(t)} = (v_{i,1}, \dots, v_{i,n})^T$  according to the position of the best particle  $\mathbf{x}_{best}^{(t)}$  towards the more promising regions in the search space. However, this movement is also dependent on the local best position of each particle  $\mathbf{p}_i^{(t)}$  and can mathematically be expressed, as follows:

$$\mathbf{v}_{i}^{(t+1)} = \mathbf{v}_{i}^{(t)} + c_{1}r_{1}^{(t)} \left(\mathbf{p}_{i}^{(t)} - \mathbf{x}_{i}^{(t)}\right) + c_{2}r_{2}^{(t)} \left(\mathbf{x}_{best}^{(t)} - \mathbf{x}_{i}^{(t)}\right).$$
(1)

Then, the new particle position is calculated according to the expression

$$\mathbf{x}_{i}^{(t+1)} = \mathbf{x}_{i}^{(t)} + \mathbf{v}_{i}^{(t)}.$$
(2)

Pseudo-code of the PSO algorithm is illustrated in Algorithm 1.

<b>Input:</b> PSO population of particles $\mathbf{x}_i = (x_{i,1}, \dots, x_{i,n})^T$ for $i = 1 \dots Np$ .			
<b>Output:</b> The best solution <b>x</b> <sub>best</sub> and its corresponding			
value $f_{min} = \min(f(\mathbf{x}))$ .			
1: init_particles;			
2: $eval = 0$ ;			
<ol><li>While termination_condition_not_meet do</li></ol>			
4: <b>for</b> $i = 1$ to <i>Np</i> <b>do</b>			
5: $f_i$ = evaluate_the_new_solution ( $\mathbf{x}_i$ );			
6: $eval = eval + 1;$			
7: <b>if</b> $f_i \leq pBest_i$ <b>then</b>			
8: $\mathbf{p}_i = \mathbf{x}_i$ ; <i>pBest</i> <sub>i</sub> = $f_i$ ; // save the local best			
solution			
9: end if			
10: <b>if</b> $f_i \leq f_{min}$ <b>then</b>			
11: $\mathbf{x}_{best} = \mathbf{x}_i; f_{min} = f_i;$ // save the global best			
solution			
12: end if			
13: $\mathbf{x}_i$ = generate_new_solution ( $\mathbf{x}_i$ );			
14: end for			
15: end while			

After finishing the initialization in function 'init\_particles' (Algorithm 1), the main loop starts in which the evaluation of fitness function takes place ('evaluate\_the\_new\_solution' function) and the new position of particle is calculated according to Eqs. (1) and (2) ('generate\_new\_solution' function). The former determines the quality of solution, while the latter moves the particle to a new possible better region of the search space [32,31]. However, between both functions the code for saving the local best as well as global best solution is executed. Note that the main loop is terminated when the termination condition (in line 3) is satisfied. Usually, the number of generation *MAX\_GEN* is used for these purposes. Over the past years, also some PSO variants were proposed [12,44,42,24,41,22,6,27].

Particle swarm optimization was also used in many practical and real-world applications. It was used in electromagnetics applications [7], spam detection [43], electric power systems [1] and many more. On the other hand, PSO was also a good candidate for solving problem of traveling salesman person [39] which is a problem of discrete optimization.

# 3.1. Applying particle swarm optimization

The generation of characters is defined as a constraint satisfaction problem (CSP), where the proper solution is found, when the conjunction of constraints  $\phi(\mathbf{x}) =$  $\wedge_{i=1...m}C_i(\mathbf{x})$  is true. Here, the graphic characters are divided into integral parts that are saved into a set of parts. Each part can be colored with different colors and can include various strokes, i.e., a shape outline with stroke width, line style and color. The former has an impact on characters look, while the later on a its shape. Generally, a development of automatic generation of graphic characters can be divided into three phases:

- Construction of the base character,
- Optimizing the modifications of the base character with the PSO algorithm,
- Generation of graphic characters from the solutions.

As an example, an automatic generation of snowman graphics was applied in this paper. The motivation for generation snowman graphics was that these graphics are relatively easy to construct, while all problems with which a designer is confronted by this generation can be identified. In the remainder of the paper generation of snowman graphics in the light of the mentioned problems are illustrated in details.

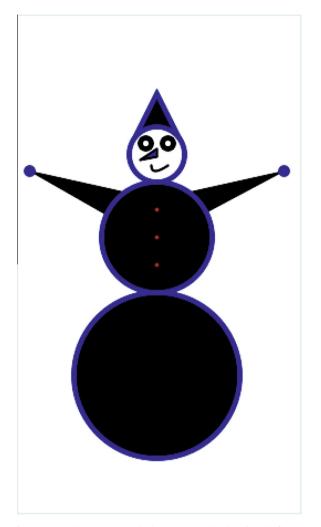
# 3.1.1. Construction of the base character

In the first phase, the base character is constructed that is then divided into integral parts. These parts that can be a subject of modification constitutes a set of parts and share the same characteristics (e.g., that are colored with the same color). However, the size of this set depends on the complexity of graphic characters. More complex the graphic character, greater the number of parts and more complex the shape.

For example, a snowman in Fig. 1 consists of 20 parts and 4 strokes, as follows: the lower part, the middle part, the upper part, the head, the hat, the left and right hand. The greater parts, like the lower, middle, upper parts and the hat are outlined with a stroke of various thickness. Each hand completes with the palm. The interior of the middle part of the snowman hull is filled with three buttons, while the head consists of two eyes, nose and mouth. The eyes and the nose are trimmed with strokes of fixed thickness, while the mouth are fixed.

#### 3.1.2. Optimization of modifications of the base character

This phase can be performed using different optimization algorithms. In this study, the PSO algorithm was applied, where each modifications of the base characters are represented as a vector  $\mathbf{x}_i = \{x_{i,1}, \ldots, x_{i,n}\}$  for  $i = 1 \dots Np$  of length n = 24 with real-valued elements in the interval  $x_{i,j} \in [0, 1]$ . The elements  $x_{i,1}, \ldots, x_{i,16}$  represent the color of defined part, and tuples  $\langle x_{i,17}, x_{i,18} \rangle, \ldots, \langle x_{i,23}, x_{i,24} \rangle$  the strokes of four greater parts of snowman. Here, the even elements of tuples denote colors, while



**Fig. 1.** A simple snowman, which we use as the benchmark for our theory. We have used the snowman for its simple shape to demonstrate the correctness of our method. Moreover, we have used neutral colors to highlight the parts that are subject to change.

#### Table 1

An example of a simple encoding scheme for colors and widths of the character.

Lower bound	Upper bound	Colors $(\Delta)$	Widths
0.0	0.1	Orange	1
0.1	0.2	Black	2
0.2	0.3	Blue	3
0.3	0.4	Red	4
0.4	0.5	Yellow	5
0.5	0.6	Green	6
0.6	0.7	Magenta	7
0.7	0.8	Cyan	8
0.8	0.9	Aqua	9
0.9	1.0	Coral	10

odd widths. Mapping of the colors and widths is implemented according to encoding scheme as represented in Table 1. Note that as higher the number of widths, the thicker the stroke.

The automatic generation of graphic characters is the CSP problem, where the proper solution is found, when

the feasibility condition  $\phi(\mathbf{x}) = true$ . Using the fitness function, the feasibility condition is expressed as

$$f(\mathbf{x}) = \sum_{i=1}^{m} \chi(C_i(\mathbf{x})), \tag{3}$$

where

$$f(\mathbf{x}) = \begin{cases} 1 & \text{if } C_i(\mathbf{x}) \text{ is false,} \\ 0 & \text{otherwise.} \end{cases}$$
(4)

In line with this, the fitness function counts the number of constraint violations. Normally, the constraints are defined by designer and depends on users demands and desires. In case of snowman characters, the following constraints were defined:

• The number of colors needed for coloring the snowman parts must be more or equal to 4. This constraint can be formally described as  $|S_1| \ge 4$ , where the set  $S_1$  is expressed as

$$S_1 = \{ \forall x_i \forall x_j | \Delta x_i \neq \Delta x_j \text{ for } i = 1 \dots 16 \land j = 1 \dots 16 \land i \neq j \},\$$

where  $\Delta$  denotes a color of the parts  $x_i$  and  $x_j$  as illustrated in Table 1.

• There must be at least 4 different strokes. This constraint can be formally described as  $|S_2| \ge 4$ , where the set  $S_2$  is defined as

$$S_2 = \{ \forall x_i \forall x_j | \Delta x_i \neq \Delta x_j \text{ for } i = 17 \dots \\ 24 \land j = 17 \dots 24 \land i \neq j \land (i \mod 2) = 1 \}.$$

• At least one stroke must have a value more than 5. Mathematically, this constraint can be written as  $|S_3| > 5$ , where the set  $S_3$  is defined as

$$S_3 = \{ \exists x_i | \Delta x_i > 5 \text{ for } i = 17 \dots 24 \land (i \mod 2) = 1 \}.$$

At least one stroke must have a value less than 5. The constraint can mathematically be expressed as |S<sub>4</sub>| < 5, where the set S<sub>4</sub> is defined as

$$S_4 = \{ \exists x_i | \Delta x_i < 5 \text{ for } i = 17 \dots 24 \land (i \mod 2) = 1 \}.$$

• The color of snowman eyes and the color of the snowman face cannot have the same color. This constraint can directly be written as a conjunction of constraints, in other words  $\Delta x_{14} \neq \Delta x_6 \wedge \Delta x_{16} \neq \Delta x_6$ , where  $x_{14}$  and  $x_{16}$  denotes eyes and  $x_6$  the snowman face (the upper part of snowman torsion).

Using the described representation of solutions evaluated by the fitness function represented in Eq. (3), the original PSO algorithm as illustrated in Algorithm 1 can be applied to the problem for automatic generation of graphic characters.

## 3.1.3. Generation of graphic characters

The solution generated by the PSO algorithm describes how to combine the colors and strokes of the snowman parts to get the unique visual presentation of 2D graphic character. Unfortunately, this description has nothing to do with the real graphical presentation of 2D snowman.

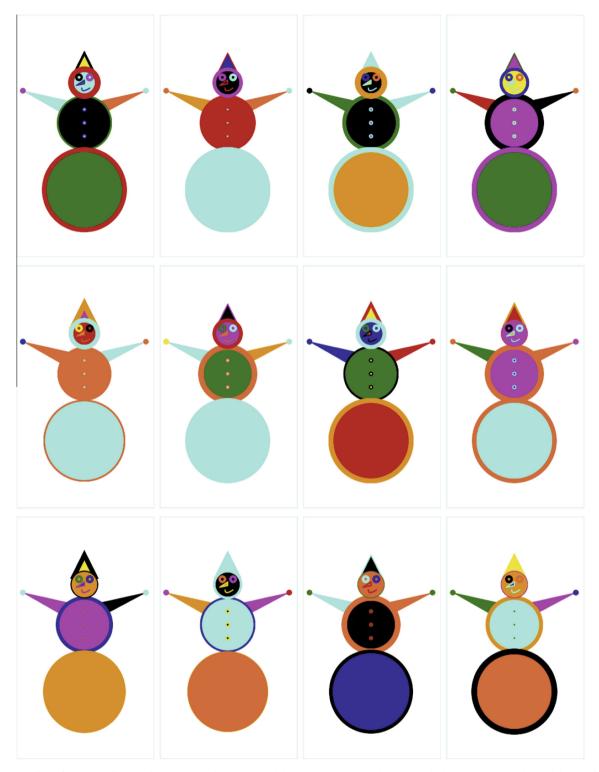


Fig. 2. A gallery of automatically created snowman, made with the particle swarm optimization algorithm. Only those snowman are depicted that satisfy all the criteria in the constraints set. The selection of the best graphic is, however, left to the judgment of the developer or user.

In order to plot the real 2D snowman graphic, an interpreter in Ruby programming language [13] was developed. Modifications can be directly transformed to call of graphical function from RMagick library [35,3]. Moreover, Ruby enables performing any operation tackling the snowman graphic characters within its script. Because the interpreter is easy to implement, the detailed description of it is left out of the scope of this paper.

#### 4. Experiments and results

The goal of our experimental work was to prove that the proposed method for automatic generation of the graphic 2D characters with the PSO algorithm could be applied to real-world video game development as well. In line with this, the set of graphical snowman parts was defined. On that basis, the PSO algorithm was developed for solving the constraint satisfaction problem of automatic generation of snowman 2D graphics. The solution of the problem was found when all constraints were satisfied as posed by designer. Finally, the proper solutions were added to a gallery of snowman graphic characters.

In the experiments, an implementation of classical PSO algorithm in Ruby was taken from author's Github repository [5]. The parameter settings of the PSO algorithm was during experiments, as follows. The values of velocities were limited to the interval  $v_i \in [-1.1]$ , the population size was set to Np = 10, maximal velocity to  $v_{max} = 100$ , and the constriction coefficients to  $c_1 = c_2 = 2.0$ . As a termination condition, the number of generation 100 was set in the case that the proper solution cannot be found. The small population size is applied because of the faster convergence of the PSO algorithm.

A gallery of properly generated snowman graphic characters is illustrated in Fig. 2 from which it can be observed that a lot of different solutions was obtained by the PSO algorithm. What is the best solution here? The running cycle of this PSO algorithm is slightly different from runs of the classical stochastic algorithms (like EA and SI), where normally the best solution is taken after satisfying the specific termination condition, (i.e., after the maximum number of generation, the maximum number of function evaluations, etc.). The best solutions in more independent runs are then evaluated according to statistical measures, as minimum, maximum, average, and standard deviation values.

The characteristics of this problem are that the solution can be found quickly. On the other hand, these solutions are not the best solutions per se. Each of the proper solutions can be viewed as the best depending on the observer's point of view. Unfortunately, this point of view can be very subjective. Therefore, all the proper solutions are aggregated into a gallery and evaluated later by ordinary users, i.e., video game developers. The video game developers are the only definitive point of reference for this assessment because they know also the context in which the generated characters are supposed to act.

# 5. Discussion

Computer games are at the heart of mass entertainment, driving not just hardware development, but pushing also the limits of software development. In fact, nowhere else are hardware and software development so closely interrelated. Video games are becoming increasingly sophisticated, and the need is clearly there to aid this development with nature-inspired algorithms. Clearly, no man-made system achieves the complexity seen in nature, so the next best thing is to learn from nature to aid artificial design.

The main contribution of our paper to this area is to help game designers facilitate the difficult task of designing large groups of characters that arise usually in historical, sports, and war-game settings. A new, highly efficient method for automatics generation of graphic characters has been proposed based on the particle swarm optimization algorithm, which is able to generate characters of arbitrary complexity. The results presented confirm that we are on the right path towards improving existing algorithms, as well as harvesting the knowledge of complexity science for use in the entertainment industry.

On a related note, which ought to be of interest to individuals working on complex systems, collective behavior and cognition, the question is if engineers looking for the best algorithms to design computer games benefit from the swarm intelligence, does it make sense to imagine that research work of the same type may help us to deepen our understanding of the origin of intelligence and cognition? [37,25,20]. While it is impossible at this point to make arguments precise, at least the conceptual link between particle swarm optimization and other nature-inspired algorithms and the emergence of intelligence is evidently there. In fact, the very use of these algorithms leads one to conclude that we are using intelligence to create new intelligence, and it may be along these lines that inspiring new discoveries could be made.

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